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Universal scaling in Bardeen–Cooper–Schrieffer superconductivity in two dimensions in non-s waves

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Abstract. The solutions of a renormalized BCS model are studied in two space dimensions for s, p and d waves for finite-range separable potentials. The gap parameter, the critical temperature T_c, the coherence length ξ and the jump in specific heat at T_c as a function of the zero-temperature condensation energy exhibit universal scalings. In the weak-coupling limit, the present model yields a small ξ and large T_c, appropriate for high-T_c cuprates. The specific heat, penetration depth and thermal conductivity as functions of temperature show universal scaling for p and d waves.

1. Introduction

At low temperature and in the weak-coupling limit, a collection of weakly interacting electrons spontaneously form large overlapping Cooper pairs [1] leading to the microscopic Bardeen–Cooper–Schrieffer (BCS) theory of superconductivity [2, 3]. For usual superconductors, the s-wave BCS theory yields $\xi k_F \sim 1000$ in agreement with experiments, where ξ is the coherence length and k_F the Fermi momentum. There has been renewed interest in this problem since the discovery of high-T_c superconductors. The high-T_c materials have a small ξ: $\xi k_F \sim 10$ [4, 5]. In spite of much effort, the normal state of the high-T_c superconductors has not been satisfactorily understood [6]. There are controversies about the appropriate microscopic Hamiltonian, pairing mechanism and gap parameter [4, 5, 7].

Many high-T_c superconductors have a conducting structure similar to a two-dimensional layer of carriers [4, 7, 8], which suggests the use of two-dimensional models. Moreover, there is evidence that the high-T_c cuprates have singlet d-wave Cooper pairs and that the gap parameter has $d_{x^2-y^2}$ symmetry in two dimensions [7]. Recent measurements of the penetration depth $\lambda(T)$ [9] and superconducting specific heat at different temperatures T [10] and related theoretical analyses [11, 12] also support this point of view. According to the isotropic s-wave BCS theory, as $T \to 0$, both observables exhibit exponential dependence on T [3, 12]. The experimental power-law dependence of these observables on T can be explained by considering an anisotropic gap parameter with node(s) on the Fermi surface for higher partial waves [11, 12]. It seems that some properties of the high-T_c cuprates can be explained by the d-wave BCS equation in two dimensions for weak coupling. Yet there has been no systematic study of the BCS equation for non-s waves in two dimensions.

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The BCS theory considers N electrons of spacing L, interacting via a weak potential of short range r_0 such that $r_0 \ll L$ and $r_0 \ll R$ where R is the pair radius. When suitably scaled, most properties of the system should be insensitive to the details of the potential and be universal functions of the dimensionless variable L/R [13]. The usual BCS treatment [2] employs a phonon-induced two-electron potential of moderate range. In this paper we study the weak-coupling BCS problem in two dimensions for s, p and d waves with two objectives in mind. The first objective is to identify the universal nature of the solution and its relation to high-T_c superconductors. The second objective is to find out to what extent the universal nature of the solution is modified in the presence of realistic finite-range (non-local separable) potentials. Instead of solving the BCS equation for the lattice with appropriate symmetry, we solved the equations for the continuum. This procedure should suffice for present objectives.

In place of the phonon-induced BCS model we employ a renormalized BCS model with a separable potential, which has certain advantages. The renormalized BCS model leads to convergent results even in the absence of potential form factors or a momentum/energy cut-off, as required in the standard BCS model. The original BCS model yields a linear correlation between T_c and T_D, where T_D is the Debye temperature. This correlation was fundamental in explaining the observed isotope effect for conventional superconductors. The high-T_c materials exhibit an anomalously negligible isotope effect and a linear correlation between T_c and T_F, where T_F is the Fermi temperature. This suggests a different interaction for superconductivity in the high-T_c materials. The present renormalized model yields a linear scaling between T_c and T_F. Because of this scaling with T_F, unlike in the phonon-induced BCS model, the present T_c can be large and appropriate for the high-T_c cuprates in the weak-coupling region. The present model also produces an appropriate T_c/T_F ratio and a small ξk_F in the weak-coupling region in accord with recent experiments [8] on high-T_c cuprates. In spite of these results, we are aware that there are controversies in the description of the high-T_c materials—for example, as regards the microscopic Hamiltonian and the pairing mechanism. Also, the normal state away from T_c seems to be very different from a standard Fermi liquid. However, we find that there are certain characteristics of these high-T_c materials which can be studied within the present renormalized mean-field BCS model based on the standard Fermi liquid theory.

Previously, there have been studies of this problem in two dimensions in terms of two-body binding in vacuum and Cooper pair binding employing short- and zero-range potentials [4, 5, 14]. Such studies have not fully revealed the universal nature of the solution. In the present paper we employ the zero-temperature condensation energy per particle, ΔU, of the BCS condensate as the reference variable for studying the BCS problem. As the condensation energy increases, one passes from weak to medium coupling. Also, ΔU is a physical observable and is the appropriate reference variable, as we shall see. In this paper we calculate the zero-temperature gap parameter $\Delta(0)$, the critical temperature T_c and the specific heat per particle $C_s(T_c)$ for all of the partial waves (characterized by the angular momentum l) and the zero-temperature pair size for the s wave. We find that these observables obey robust universal scaling as functions of ΔU, valid over several decades in the weak-to-medium-coupling region, independently of the potential range parameter.

Here, we also calculate the temperature dependences of different quantities, such as the gap parameter $\Delta(T)$, $C_s(T)$, $\lambda(T)$ and the thermal conductivity $K(T)$ for $T < T_c$. Of these, the T-dependences of $C_s(T)$, $\lambda(T)$ and $K(T)$ are especially interesting. For isotropic s waves, the BCS theory yields an exponential dependence on temperature as $T \to 0$ for these observables independently of the space dimension [3, 12]. The observed power-law dependence of some of these observables can be explained with an anisotropic gap parameter for non-s waves with nodes on the Fermi surface. We find a universal power-law dependence in both cases for non-s waves independently of the range of the potential. For $l \neq 0$ we find $C_s(T) \approx 2C_n(T_c)(T/T_c)^2$ and $K_s(T) \approx K_n(T)(T/T_c)^{1.2}$ to be valid for almost the entire range of temperature. The subscripts n and s refer to normal and superconducting states, respectively. Similar dependences were predicted from an analysis of experimental data [10] as well as from a calculation based on the Eliashberg equation [11]. In order to detect the anisotropy in $\Delta(T)$ it is appropriate to consider the function $\Delta\lambda \equiv (\lambda(T) - \lambda(0))/\lambda(0)$ [12]. We find that for small T it behaves as $\Delta\lambda \sim (T/T_c)^{1.3}$. A similar power-law dependence was conjectured earlier [12].

From the weak-coupling BCS equation we establish the following relations analytically: $2\Delta(0)/T_c \approx 3.528$ (3.026), $\Delta(0) = 2\sqrt{\Delta U}$ ($2\sqrt{\Delta U}$), $T_c \approx 1.134\sqrt{\Delta U}$ ($1.322\sqrt{\Delta U}$), $\Delta C/C_n(T_c) \approx 1.43$ (1.05), $C_s(T_c) \approx 9.065\sqrt{\Delta U}$ ($8.915\sqrt{\Delta U}$), $\Delta U/U_n(T_c) \approx 0.473$ (0.348), for $l = 0$ ($l \neq 0$). Here ΔC is the jump in specific heat at $T = T_c$ and C (U) is the specific heat (internal energy) per particle.

The plan of the paper is as follows. In section 2 we present the renormalized BCS model. In section 3 we present an account of our numerical study. Finally, in section 4 we present a brief summary of our findings.

2. The renormalized BCS model

The two-body problem and the Cooper and BCS models all exhibit ultraviolet divergences for zero-range potentials and require regularization or renormalization to produce finite results [4]. The usual BCS model employs a cut-off regularization (the Debye cut-off) for obtaining finite observables. In momentum space, the standard BCS model interaction is taken to be constant for momenta between $2m(E_F - E_D)/\hbar^2$ and $2m(E_F + E_D)/\hbar^2$ and zero elsewhere with E_D (E_F) the Debye (Fermi) energy. This implies a moderate range of the interaction. This potential is a physically motivated phonon-induced electron–electron one [2]. The present renormalized BCS equation may lead to a well-defined solution without requiring a cut-off even in the absence of potential form factors. The present model with a finite-range potential also leads to a well-defined mathematical problem.

We consider a two-body system, with each body of mass m, in the centre-of-mass frame [4]. The single- (two-) particle energy is given by $\epsilon_q = \hbar^2 q^2/2m$ ($2\epsilon_q = \hbar^2 q^2/2m$), where q is the wavenumber. We consider a purely attractive short-range separable potential in terms of the partial wave l:

$$V_{\boldsymbol{pq}} = -V_0 c_l g_{pl} g_{ql} \cos(l\theta) \tag{1}$$

where θ is the angle between the vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ and $c_l = 1$ (2) for $l = 0$ ($l \neq 0$) as in reference [15]. Here g_{pl} and g_{ql} are potential form factors and V_0 is the potential strength. The potential $V_{\boldsymbol{pq}}$ is the effective electron–electron potential in the superconductor in the presence of lattice and other electrons. The Schrödinger equation in this case leads to the following condition for a two-particle bound state in vacuum with binding B_2:

$$V_0^{-1} = c_l \sum_{\boldsymbol{q}} g_{ql}^2 \cos^2(l\theta)(B_2 + 2\epsilon_q)^{-1} \tag{2}$$

where θ is the angle of the vector $\boldsymbol{q}$ and $\epsilon_q = \hbar^2 q^2/2m$ with m the mass. In order to simplify the notation, the trivial l-dependence of many quantities, such as $V_{\boldsymbol{pq}}$ and B_2, is suppressed.

For the attractive potential (2), the existence of two electrons on the top of the full Fermi sea at zero temperature shows pairing instability. Singlet (triplet) pairing is assumed to take place in even (odd) partial waves. The Cooper pair problem for two electrons above the filled Fermi sea for this potential is given by [1, 2] $V_0^{-1} = c_l \sum_{q>1} g_{ql}^2 \cos^2(l\theta)(2\epsilon_q - 2\hat{E})^{-1}$, with the Cooper binding $B_c \equiv 2 - 2\hat{E}$. Using (2), the Cooper problem is written as

$$\sum_q g_{ql}^2 \cos^2(l\theta)(B_2 + 2\epsilon_q)^{-1} - \sum_{q>1} g_{ql}^2 \cos^2(l\theta)(2\epsilon_q - 2\hat{E})^{-1} = 0. \tag{3}$$

At a finite T, one has the following BCS and number equations:

$$\Delta_{\boldsymbol{p}} = -\sum_{\boldsymbol{q}} V_{\boldsymbol{pq}} \frac{\Delta_{\boldsymbol{q}}}{2E_{\boldsymbol{q}}} \tanh \frac{E_{\boldsymbol{q}}}{2T} \tag{4}$$

$$N = \sum_{\boldsymbol{q}} \left[1 - \frac{\epsilon_q - \mu}{E_{\boldsymbol{q}}} \tanh \frac{E_{\boldsymbol{q}}}{2T} \right] \tag{5}$$

with $E_{\boldsymbol{q}} = [(\epsilon_q - \mu)^2 + |\Delta_{\boldsymbol{q}}|^2]^{1/2}$, where $\Delta_{\boldsymbol{q}}$ is the gap function. Unless the units of the variables are explicitly mentioned, in (2)–(5) and in the following, all energy/momentum variables are expressed in units of E_F, such that $\mu \equiv \mu/E_F$, $T \equiv T/T_F$, $q \equiv q/k_F$, $E_{\boldsymbol{q}} \equiv E_{\boldsymbol{q}}/E_F$ etc, where μ is the chemical potential. Here $\Delta_{\boldsymbol{q}}$ has the following anisotropic form: $\Delta_{\boldsymbol{q}} \equiv g_{ql} \Delta_0 \sqrt{c_l} \cos(l\theta)$ where Δ_0 and g_{ql} are dimensionless. The usual BCS gap is defined by $\Delta(T) = g_{q(=1)l}\Delta_0$, which is the root mean square average of $\Delta_{\boldsymbol{q}}$ on the Fermi surface. Using the above form of $\Delta_{\boldsymbol{q}}$, the BCS equation (4) can be written as

$$\frac{1}{V_0} = c_l \sum_{\boldsymbol{q}} g_{ql}^2 \cos^2(l\theta) \frac{1}{2E_{\boldsymbol{q}}} \tanh \frac{E_{\boldsymbol{q}}}{2T}. \tag{6}$$

Now (2) and (6) lead to

$$\sum_{\boldsymbol{q}} g_{ql}^2 \cos^2(l\theta) \left[\frac{2}{2\epsilon_q + B_2} - \frac{1}{E_{\boldsymbol{q}}} \tanh \frac{E_{\boldsymbol{q}}}{2T} \right] = 0. \tag{7}$$

The summation is evaluated according to

$$\sum_{\boldsymbol{q}} \to \frac{N}{4\pi} \int \int \mathrm{d}\epsilon_q \, \mathrm{d}\theta \equiv \frac{N}{4\pi} \int_0^\infty \mathrm{d}\epsilon_q \int_0^{2\pi} \mathrm{d}\theta \tag{8}$$

where N is the number of electrons. As in the standard BCS model, we have a constant density of states in (8) but now with the generalization to include the angular dependence. With the help of (8), equations (5) and (7) can be explicitly written as

$$\int \int \mathrm{d}\epsilon_q \, \mathrm{d}\theta \left[1 - \frac{\epsilon_q - \mu}{E_{\boldsymbol{q}}} \tanh \frac{E_{\boldsymbol{q}}}{2T} \right] = 4\pi \tag{9}$$

$$\int \int \mathrm{d}\epsilon_q \, \mathrm{d}\theta \; g_{ql}^2 \cos^2(l\theta) \left[\frac{2}{2\epsilon_q + B_2} - \frac{1}{E_{\boldsymbol{q}}} \tanh \frac{E_{\boldsymbol{q}}}{2T} \right] = 0 \tag{10}$$

respectively. In terms of Cooper-pair binding, equation (10) can be rewritten as

$$\int_0^{2\pi} \mathrm{d}\theta \cos^2(l\theta) \left[\int_1^\infty \mathrm{d}\epsilon_q \, \frac{g_{ql}^2}{\epsilon_q - \hat{E}} - \int_0^\infty \mathrm{d}\epsilon_q \, \frac{g_{ql}^2}{E_{\boldsymbol{q}}} \tanh \frac{E_{\boldsymbol{q}}}{2T} \right] = 0. \tag{11}$$

Even in the absence of the potential form factors $g_{pl} = 1$, equations (9), (10) and (11) are well defined without any energy/momentum cut-off, though each part of the integral in these equations diverges separately. This is why this model is termed 'renormalized'. Potential (1) with $g_{pl} = 1$ is the zero-range delta-function potential. The standard BCS model uses essentially the above potential with an energy/momentum cut-off for obtaining convergence. In the renormalized equation (10), the usual energy/momentum cut-off and the potential strength V_0 have been eliminated in favour of the two-body binding B_2. Recently, the role of renormalization in non-relativistic quantum mechanics has been discussed [16].

Equation (11) has the following analytic solutions for the s-wave zero-range potential ($g_{q0} = 1$) in the weak-coupling limit ($\mu = 1$). At $T = 0$, $\Delta(0) = \sqrt{2B_c}$. At $T = T_c$ ($\Delta = 0$), we have $T_c = \exp(\gamma)\sqrt{2B_c}/\pi \approx 0.8\sqrt{B_c}$ where $\gamma = 0.577\,22\ldots$. The standard BCS model yields in this case $T_c/T_D = \exp(\gamma)\sqrt{2B_c}/\pi\sqrt{T_D} \approx 0.8\sqrt{B_c/T_D}$ where T_D is the Debye temperature. To illustrate the advantage of the renormalized model, let us consider a specific example with $T_D = 300$ K, $T_F = 3000$ K and $B_c = 10$ K. We take $B_c < 10$ K as defining the weak-coupling region. With $B_c = 10$ K, the standard BCS model yields $T_c = 44$ K, whereas the present renormalized model yields $T_c = 138$ K. Hence for the same coupling, the renormalized model leads to an enhanced T_c. In the standard BCS model one has to have a much larger B_c, clearly outside the weak-coupling region, in order to have $T_c > 100$ K.

The universal ratio $2\Delta(0)/T_c = 2\pi/\exp(\gamma) \approx 3.528$ remains unchanged for the s wave in three dimensions as well as for the trivial case of anisotropic pairing ($\sim \exp(\mathrm{i}l\theta)$) in two dimensions for $l \neq 0$ [14]. In reference [14] we essentially changed the potential form factors without introducing any explicit angular dependence in the BCS and number equations and found that the universal nature of the solution was unchanged on making such a change. Hence we expect to extract certain universal properties of (10) analytically for $l \neq 0$ by employing the correct angular distribution and no potential form factors ($g_{ql} = 1$).

Next we study (10) analytically for weak coupling ($\mu = 1$, $\Delta \ll 1$) and for $g_{ql} = 1$. At $T = 0$, equation (10) can be integrated to yield

$$\int_0^{2\pi} \mathrm{d}\theta\ \cos^2(l\theta) \ln \frac{[1 + \Delta^2(0) c_l \cos^2(l\theta)]^{1/2} - 1}{B_2} = 0$$

which for $\Delta(0) \ll 1$ and $l \neq 0$ reduces to

$$\ln \frac{c_l \Delta^2(0)}{2B_2} = -\frac{1}{\pi} \int_0^{2\pi} \mathrm{d}\theta\ \cos^2(l\theta) \ln \cos^2(l\theta) \equiv 0.3863$$

or $\Delta(0) \approx 1.213\sqrt{B_2}$. At $T = T_c$, we again have $T_c = \exp(\gamma)\sqrt{2B_2}/\pi$, so we have the following new universal constant for $l \neq 0$: $2\Delta(0)/T_c \approx 3.026$.

The condensation energy per particle at $T = 0$ is defined by [3]

$$\Delta U \equiv |U_s - U_n| = \frac{1}{N} \sum_{\boldsymbol{q}\,(q<1)} 2\zeta_q - \frac{1}{N} \sum_{\boldsymbol{q}} \left(\zeta_q - \frac{\zeta_q^2}{E_{\boldsymbol{q}}} - \frac{\Delta_{\boldsymbol{q}}^2}{2E_{\boldsymbol{q}}} \right)$$

where $\zeta_q = (\epsilon_q - \mu)$. This can be evaluated straightforwardly to lead to [3]

$$\Delta U = \frac{1}{8\pi} \int_0^{2\pi} c_l \Delta^2(0) \cos^2(l\theta)\ \mathrm{d}\theta$$

which yields $\Delta(0) = 2\sqrt{\Delta U}$ for all l. Using the universal relation between $\Delta(0)$ and T_c given above, one has $T_c \approx 1.134\sqrt{\Delta U}$ ($1.322\sqrt{\Delta U}$) for $l = 0$ ($l \neq 0$). For all l, $U_n(T_c) = \pi^2 T_c^2/6$, so $\Delta U/U_n(T_c) \approx 0.473$ (0.348) for $l = 0$ ($l \neq 0$).

The superconducting specific heat per particle is given by

$$C_s = \frac{2}{NT^2} \sum_{\boldsymbol{q}} f_{\boldsymbol{q}}(1 - f_{\boldsymbol{q}}) \left(E_{\boldsymbol{q}}^2 - \frac{1}{2} T \frac{\mathrm{d}\Delta_{\boldsymbol{q}}^2}{\mathrm{d}T} \right) \tag{12}$$

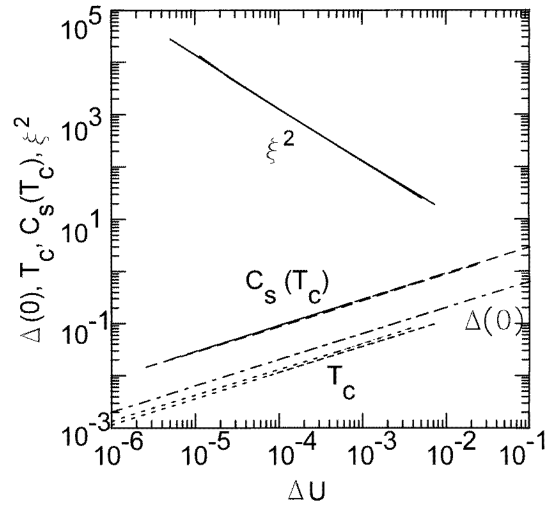

Figure 1. Plots of the specific heat $C_s(T_c)$ (dashed line), T_c (dotted line) and gap parameter $\Delta(0)$ (chain line) for s, p and d waves, and the s-wave pair radius ξ^2 (solid line) versus the zero-temperature condensation energy per particle ΔU for different potential parameters for α between 1 and ∞. For the first two variables there are two distinct lines: the upper one is for p and d waves and the lower one is for s waves.

where $f_q = 1/(1+\exp(E_q/T))$. The normal specific heat C_n is given by (12) with $\Delta_q = 0$. The jump in specific heat per particle at $T = T_c$ ($\Delta(T_c) = 0$), $\Delta C \equiv [C_s - C_n]_{T_c}$, is given by [3]

$$\Delta C = -\frac{1}{NT_c}\sum_q \left[f_q(1-f_q)\frac{\mathrm{d}\Delta_q^2}{\mathrm{d}T} \right]_{T_c}. \tag{13}$$

In the special case where $g_{ql} = 1$, the radial integral in (13) can be evaluated as in reference [3] and we get

$$\Delta C = -\int c_l \; \frac{\mathrm{d}\epsilon_q \; \mathrm{d}\theta}{4\pi T_c} \left[f_q(1-f_q)\frac{\mathrm{d}\Delta^2(T)}{\mathrm{d}T} \right]_{T_c} \cos^2(l\theta). \tag{14}$$

This leads to [3] $\Delta C = -(1/2)[\mathrm{d}\Delta^2(T)/\mathrm{d}T]_{T=T_c}$ for all l. In a systematic (numerical) study we find $\Delta(T)/\Delta(0) = B(1 - T/T_c)^{1/2}$ valid for $T \approx T_c$, with $B \approx 1.74$ (1.70) for $l = 0$ ($l \neq 0$). Using the value of B and the universal ratio $2\Delta(0)/T_c$, we obtain $\Delta C/C_n(T_c) \approx 1.43$ (1.00) for $l = 0$ ($l \neq 0$), where $C_n(T) = \pi^2 T/3$. Consequently, $C_s(T_c)/\sqrt{\Delta U} \approx 9.065$ (8.915) and $T_c C_n(T_c)/\Delta U \approx 4.229$ (5.749) for $l = 0$ ($l \neq 0$).

The penetration depth λ is defined by [3]

$$\lambda^{-2}(T) = \lambda^{-2}(0)\left[1 - \frac{2}{NT}\sum_q f_q(1-f_q) \right]. \tag{15}$$

The thermal conductivity ratio $K_s(T)/K_n(T)$ is defined by [17]

$$\frac{K_s(T)}{K_n(T)} = \left(\sum_q \zeta_q E_q f_q(1-f_q) \right) \Big/ \left(\sum_q \zeta_q^2 f_q(1-f_q) \right). \tag{16}$$

The denominator of (16) essentially corresponds to the normal-state thermal conductivity with the BCS gap set equal to zero.

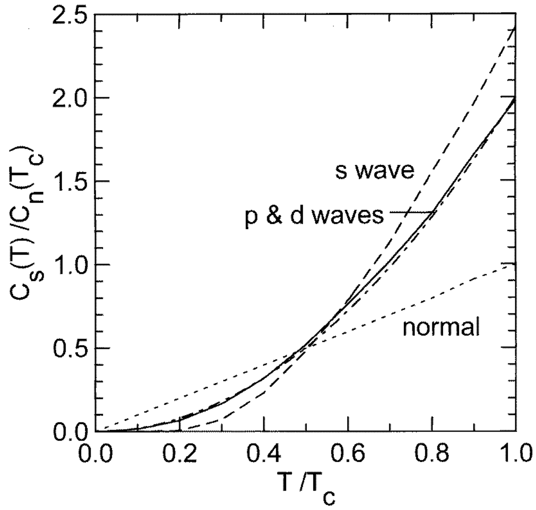

Figure 2. The specific heat $C_s(T)/C_n(T_c)$ versus T/T_c for s (dashed line), p and d (solid line) waves and potential parameters for α between 1 and ∞. The analytic fit (chain line) of $C_s(T)/C_n(T_c) = 2(T/T_c)^2$ to p and d waves is also shown.

The dimensionless s-wave pair radius defined by $\xi^2 = \langle\psi_q|r^2|\psi_q\rangle/\langle\psi_q|\psi_q\rangle$ with the pair wave function $\psi_q = g_{ql}\Delta/(2E_q)$ can be evaluated by using $r^2 = -\nabla_q^2$. In the weak-coupling limit, the zero-range analytic result of reference [4] leads to $\xi^2 = 0.5\Delta^{-2}(0) = 0.125(\Delta U)^{-1}$.

3. Numerical results

We solve the coupled equations (9) and (10) numerically using the dimensionless form factors $g_{ql} = (\epsilon_q)^{l/2}[\alpha/(\epsilon_q + \alpha)]^{(l+1)/2}$ with the correct threshold behaviour for small momenta, where α is the range parameter. Following reference [3] we calculate the dimensionless gap parameter $\Delta(0) = g_{q(=1)l}\Delta$, T_c, $C_s(T_c)$, the s-wave pair radius ξ^2 at $T = 0$ as well as $\Delta(T)$, $\lambda(T)$ and $C(T)$ for different coupling. In figure 1 we plot $\Delta(0)$, T_c, $C_s(T_c)$ and ξ^2 at $T = 0$ versus ΔU, and find universal scalings. The calculations were repeated for different potential ranges α. We varied α from 1 to ∞ and found figure 1 to be insensitive to this variation for each partial wave. For p and d waves, equations (9) and (10) diverge for $\alpha \to \infty$ and calculations were performed for $\alpha = 1$ to 10. The increase in ΔU of figure 1 corresponds to an increase in coupling. We could express this change in coupling in terms of a change in B_2 or in the Cooper pair binding and plot the variables of figure 1 in terms of these bindings as in reference [14]. Then each value of the range parameter leads to a distinct curve. However, if we express the variation in coupling in terms of a variation of an observable of the superconductor, such as ΔU or T_c, universal potential-independent scalings are obtained. In each case the exponent and the prefactor of each scaling relation are in excellent agreement with the analytic relation obtained above without form factors.

T_c should not arbitrarily increase with coupling as figure 1 may imply. With increased coupling the electron pairs should form composite bosons which may undergo a phase transition under the action of a residual interaction. According to a numerical study this transition happens at a temperature of 0.1 [18]. This is why the T_c-curve in figure 1 has been plotted up to about $T_c = 0.1$. For a very large class of two-dimensional high-T_c materials, T_c has been estimated to be about 0.05 [8], which corresponds, for $g_{ql} = 1$, to $B_2 = 0.004$. This small value of B_2 is in the weak-coupling region where the universality of the present study should hold. The corresponding dimensionless pair radius ($\xi^2 \sim 80$) at $T_c = 0.05$, as obtained from figure 1, is in agreement with experimental analysis [8]. Hence, the present study is relevant for these high-T_c materials.

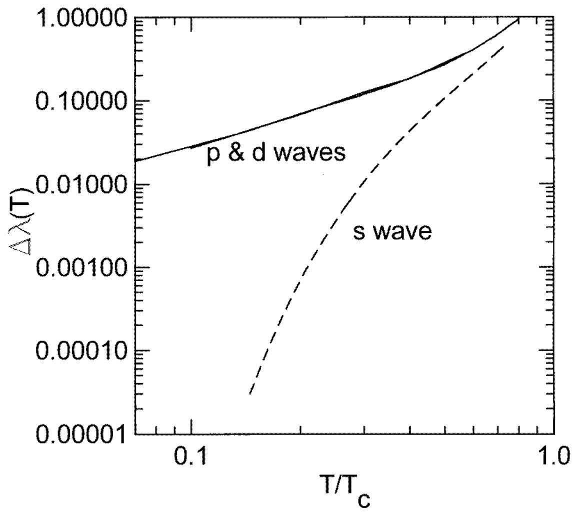

Figure 3. The penetration depth $\Delta\lambda(T)$ versus T/T_c for s (dashed line), p and d (solid line) waves and potential parameters for α between 1 and ∞.

Next we studied the temperature dependence of $\Delta(T)$, $C_s(T)$, $\lambda(T)$ and $K(T)$ for $T < T_c$. For BCS superconductors, these observables have an exponential dependence on T as $T \to 0$ [3, 12]. The two-dimensional high-T_c superconductors have a power-law dependence on T. In figures 2, 3 and 4 we plot $C_s(T)/C_n(T_c)$, $\Delta\lambda(T) \equiv (\lambda(T)-\lambda(0))/\lambda(0)$ and $K_s(T)/K_n(T)$ versus T/T_c, respectively. In these figures we find a universal power-law dependence, essentially independent of the potential range, for $l \neq 0$. We find $C_s(T) \approx 2C_n(T_c)(T/T_c)^2$, $K_s(T) \approx 2K_n(T)(T/T_c)^{1.2}$ for almost the entire temperature range and $\Delta\lambda(T)/\lambda(0) \sim (T/T_c)^{1.3}$ for small T/T_c. The T^2-dependence of $C_s(T)$ was found in a theoretical study of the Eliashberg equation [11] and in an analysis of the experimental data [10]. The power-law dependence of $\lambda(T)$ on T was also conjectured earlier [12]. The gap function $\Delta(T)$ has essentially the same universal behaviour as for the s wave [3].

4. Summary

Scalings are established among $\Delta(0)$, T_c, $C_s(T_c)$ and ξ^2, as functions of ΔU, independently of the potential range of the s, p and d waves, from a study of a renormalized BCS equation in two dimensions. The present renormalized model yields a large T_c and a small ξ appropriate for high-T_c superconductors. The T-dependences of $\Delta\lambda(T)$, $C_s(T)$ and $K_s(T)$ below T_c for non-s waves show power-law scalings appropriate for high-T_c materials at low energies. No power-law T-dependence is found for the s wave for $\Delta\lambda(T)$ and $C_s(T)$. Calculations

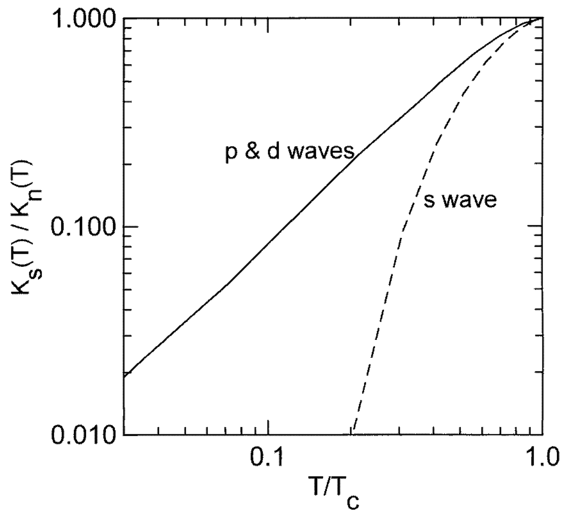

Figure 4. The thermal conductivity ratio $K_s(T)/K_n(T)$ versus T/T_c for s (dashed line), p and d (solid line) waves and potential parameters for α between 1 and ∞.

performed with $\cos(l\theta)$ and $\sin(l\theta)$ angular dependences yielded identical results. Although we have used a separable potential, in view of the universal nature of the study we do not believe the present conclusions to be so peculiar as to have no general validity. We have repeated the s-wave calculations with a local Yukawa potential and found the results to be independent of the potential in the weak-coupling region. This is also in agreement with a suggestion made by Leggett [13]. Although there are controversies about the microscopic formulation for high-T_c superconductors, it seems that the two-dimensional d-wave BCS equation for weak coupling can be used to explain some of their universal scalings. A similar study of universality has recently been performed in three dimensions employing the renormalized BCS equation [19], which also leads to an enhanced T_c in the weak-coupling limit.

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